Two Level Implicit Discretization of High Order for The Solution of 2-D Non-Linear Parabolic Partial Differential Equations of Non-Uniform Grid Points

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ABSTRACT

In this chapter, a new two-level implicit-difference method of $O(k^2 + kh^2 + h^4)$ for the solution of singularly perturbed non-linear parabolic differential equation $\varepsilon(u_{xx} + u_{yy}) = f(x, y, t, u, u_x, u_y, u_t)$, $o < x, y < 1$, $t > 0$ subject to appropriate initial and Dirichlet boundary conditions, where $k > 0$ and $h > 0$ are grid sizes in time and space directions, respectively, and $\varepsilon > 0$ is a small parameter has been reported. New methods of $O(kh^2 + h^4)$ for the estimates of $(\partial u / \partial x)$ and $(\partial u / \partial y)$ hare also been reported, where, x & y are independent variables. In all case, we use 9-spatial grid points and a single computational cell. These methods are directly applicable to singular problems. No special scheme to singular problems is required. Alternating direction implicit (ADI) method for solving diffusion equation in polar cylindrical coordinates has also been solved. This method permits multiple use of the one-dimensional tri-diagonal algorithm with a considerable solving in computing time, and produces a very efficient solver. It is shown that the ADI method is unconditionally stable. Numerical experiments are conducted to test the high accuracy of the reported methods and compared with the exact solutions.